"On a Throw-testing Machine for Reversals of Mean Stress." By Osborne Reynolds, F.R.S., and J. H. Smith, M.Sc., Whitworth Scholar, Victoria University. Received March 5, —Read March 20, 1902.

(Abstract.)

The present research, which was carried on in the Whitworth Engineering Laboratory of the Owens College, Manchester, was undertaken at the suggestion of Professor Osborne Reynolds, who proposed an investigation of "repeated stress" on the following lines :—The stress should be direct tension, and compression of approximately equal amounts, such tension and compression being obtained by means of the inertia force of an oscillatory weight. The rapidity of repetitions should be much higher than in the experiments of Wöhler, Spangenberg, Bauschinger, and Baker—in fact, ranging as high as 2000 reversals per minute.

In the apparatus employed a weight is supported vertically by means of the specimen to be tested, and the upper part of the specimen receives a periodic motion in a vertical direction by means of a crank and a connecting rod. The inertia of this weight gives a tension at the bottom end, and a compression at the top end of the stroke, the change from tension to compression being gradual. The specimen and parts are guided by suitable bearings placed in a vertical direction. The motion was made vertical in order to reduce the friction of the bearings to a minimum. The stresses can be changed by varying the diameter of the specimen, the load, and the speed of revolution of the crank. In order to enable one to calculate the stresses in the specimen, the centre of the crank shaft must be at rest, and the crank must move with uniform angular velocity. These conditions are obtained when the crank shaft is driven by a constant turning effort, if the moving parts of the machine are balanced, and if at the same time the total energy of the moving parts is invariable. The apparatus was therefore designed to satisfy these conditions as approximately as possible.

The apparatus was driven by the low-pressure engine of the triple expansion experimental engines, and had a speed indicator and a revolution counter attached. A great amount of trouble was experienced in lubricating the machine and in keeping the fluctuations of velocity small.

The specimens employed were carefully prepared, and with a few exceptions were of constant length and diameter. They were, in most cases, annealed before testing.

In conducting the tests the reversals for rupture were estimated

from the mean speed and the interval between the time of attaining full speed and breaking. In many cases the test could not be carried out without interruption, and for this reason the specimens had to rest, sometimes for a few days. The effect of these periods of rest on the total reversals for rupture was investigated, and found to be negligible.

In carrying out the tests it was found that for a series of tests in which the range of stress was being lowered (the specimens were of constant diameter), the limiting range of stress was more rapidly approached by this apparatus than by that of Wöhler. Since the diminution of range was obtained by diminishing the speed, this suggested that the limiting range of stress varied with the speed.

It is possible to use six different loads for the machine, and therefore to repeat a test with a given range of stress at six different speeds. It was observed that when more than one million reversals were required for rupture the rate of change of reversals with range of stress was very great indeed, and the author for this reason decided to limit in general the tests to one million reversals.

In the case of mild steel, six sets of tests were carried out corresponding to the six different loads which could be applied to the machine; six sets of results, similar to those of Wöhler, were obtained; these results were plotted, and the range of stress for rupture with one million reversals was obtained at six different speeds. The results were :—

Range of stress for rupture with 10^6 reversals.	Reversals per minute.
20·9	1337
20·1	1428
19·2	1516
18·1	1656
15·2	1744
12·4	1917

The mean result of statical tests for the mild steel employed in these experiments was :—

Yield stress	17·12 tons
Maximum stress	24·54 ,,
Breaking ,, ...	20·47 ,,
Percentage elongation............	30

In the case of cast steel, four sets of tests were carried out. The results obtained were very little different from those of mild steel.

Thus the important conclusions arrived at are :—

1. The reversals for rupture with a given range of stress diminishes as the periodicity of the reversals increases.

2. The hard steels will not withstand a greater number of reversals of the same range of stress than the mild steels if the periodicity of the reversals is great.

"The Equilibrium of Rotating Liquid Cylinders." By J. H. JEANS, B.A., Isaac Newton Student and Fellow of Trinity College, Cambridge. Communicated by Professor G. H. DARWIN, F.R.S. Received March 6,—Read March 20, 1902.

(Abstract.)

The most serious obstacle to progress in the problem of determining the equilibrium configurations of a rotating liquid lies in the difficulty of determining the potential of a mass of homogeneous matter of which the boundary is given. If this boundary is

$$f(x, y, z) = 0 \quad \text{(i)},$$

the potential will be a unique-valued function of x, y, and z, of which the form will depend solely upon the form of $f(x, y, z)$. This potential must therefore be deducible by some algebraical transformation of the function f.

In the method usually followed the solution is found as a volume integral, the integration extending throughout the surface (i). There is, however, a second method of obtaining this potential, namely, by regarding the potential-function as the solution of a differential equation, subject to certain boundary conditions. This leads directly to a series of algebraical processes, enabling us (theoretically) to deduce the potential by transformation of the function f.

In three-dimensional problems this method is quite impracticable, since it depends upon a continued application of the formula which expresses the products or powers of spherical harmonics as the sum of a series of harmonics.

As soon, however, as we pass to the consideration of two-dimensional problems, the spherical harmonics may be replaced by circular functions of a single variable. The transformation now becomes manageable, and for this reason the present paper deals only with two-dimensional problems, for which a method is developed enabling us to write down the potential by transformation of the equation of the boundary. The method is not of universal applicability, but is adequate to the problem in hand.

The method as applied to the determination of equilibrium configurations is as follows. Starting from the general equation (in polar co-ordinates)

$$r^2 = a_0 + 2a_1 r \cos\theta + 2a_2 r^2 \cos 2\theta + \ldots \quad \text{(ii)},$$